Complexity Theory, Semester I 2019 - IIC3242
Homework 4
Deadline: Friday, June 7th, 2019

1 Cycle [2 points]

A cycle in a *directed* graph $G = (V, E)$ is a sequence $v_0 v_1 \ldots v_k$, where $k \geq 1$, $(v_i, v_{i+1}) \in E$, for $0 \leq i \leq k-1$, and $v_0 = v_k$. Consider the language:

$$\text{CYCLE} = \{ \langle G, v \rangle \mid \text{ where } v \text{ belongs to some cycle in } G \}.$$

Show that CYCLE is NLOGSPACE-complete. Note that you need to show both the upper and the lower bound.

2 Inefficient problems [1 point]

Define the language U as follows:

$$U = \{ \langle M, w, \#^t \rangle \mid M \text{ is a non-deterministic TM which accepts } w \text{ within } 2^t$$
$$\text{steps, on some branch of its computation } \}.$$

Show that U can not be decided in polynomial time.

3 Impact of inefficient problems on PTIME and NP [3 points]

Show that 2EXPTIME $\neq$ 2NEXPTIME implies that PTIME $\neq$ NP. Recall that 2EXPTIME denotes the class of all languages solvable by a deterministic Turing machine running in time $O(2^{2^{n^c}})$, and similarly for 2NEXPTIME.

Hint: Problem 2 can help you.