PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory, Semester I 2019 - IIC3242
Homework 2
Deadline: Tuesday, 30th April 2019 (anywhere on Earth)

1 Is SAT actually PTIME? [2 points]

Consider the following language, where k is an arbitrary natural number:

$$\text{k-SAT} = \{\ \langle \varphi \rangle\ \mid\ \varphi \text{ is a valid formula with } \textbf{at most}\ k \text{ variables}\ \}.$$

Can you show that k-SAT is solvable in time $O(n^m)$ for some natural number m? Explain why or why not.

2 Isomorphic subgraphs [5 points]

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two undirected graphs. We will say that G_1 contains a subgraph $H = (V_H, E_H)$ that is *isomorphic* to G_2 if:

- $V_H \subseteq V_1$
- $E_H \subseteq E_1$
- $|V_H| = |V_2|$
- There exists a function $f : V_2 \to V_H$ such that:
  - f is injective
  - $\{v, v'\} \in E_2$ if and only if $\{f(v), f(v')\} \in E_H$.

We define the following language:

$$\texttt{SUBGRAPH ISOMORPHISM} = \{\langle\ G_1, G_2\rangle\ \mid\ G_1 \text{ contains a subgraph } H \text{ isomorphic to } G_2\}.$$

Show that `SUBGRAPH ISOMORPHISM` is NP-complete.