



PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE  
ESCUELA DE INGENIERIA  
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

**Complexity Theory, Semester I 2019 - IIC3242**

**Homework 1**

Deadline: Thursday, 11 April 2019 (anywhere on Earth)

## 1 The Goldbach problem [2 points]

Consider the following string:

$$s = \begin{cases} 1, & \text{if Goldbach's conjecture is true} \\ 0, & \text{if Goldbach's conjecture is not true} \end{cases}$$

Let  $L = \{s\}$  be the language containing only the string  $s$ . Is  $L$  decidable? Explain why or why not?

**Note:** Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states that every even integer greater than 2 can be expressed as the sum of two primes. There is still no proof for this conjecture.

## 2 Programming Turing machines [4 points]

The web page <https://turingmachinesimulator.com/> provides a programmable Turing machine. Here you can determine the number of tapes used, the vocabulary, etc. One difference with the model we considered is that the tape is infinite in both directions.

In this problem, you are asked to create a program for this Turing machine simulator that decides the following language:

$$L = \{ w \cdot \# \cdot w^{-1} \mid w \in \{0,1\}^* \text{ and } |w| \leq 4 \}.$$

Here  $w^{-1}$  denotes the reverse of the word  $w$ , and  $|w|$  denotes the length of  $w$ .

You should submit a printout of your program, preferably in .txt format. You should also give a high level description explaining the logic of your program, and what each set of states is supposed to do. Finally, give a rough analysis of the running time of your machine on an input of length  $n$  using big-O notation.