PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory, Semester I 2018 - IIC3242
Homework 5
Deadline: Wednesday, June 6, 2018

1 Easy polynomial hierarchy [3 points]

Let $\mathcal{A}$ and $\mathcal{B}$ be two finite sets of directed graphs. We say that a graph G separates $\mathcal{A}$ from $\mathcal{B}$, if there exists a graph $A \in \mathcal{A}$ that is isomorphic to some subgraph of G, and no graph $B \in \mathcal{B}$ is isomorhic to some subgraph of G. Recall that a graph A is isomorphic to a subgraph of G, if there is a function $f : V(A) \mapsto V(G)$, where $V(A)$ denotes the vertices of A and $V(G)$ the vertices of G, such that (a, b) is an edge in A if and only if $(f(a), f(b))$ is an edge in G, for all $a, b \in V(A)$. Consider the following language:

$$L = \{\ (\mathcal{A}, \mathcal{B}, k)\ \mid\ \text{there is a graph } G = (V, E) \text{ separating } \mathcal{A} \text{ from } \mathcal{B} \text{ and } |V| \leq k\ \}.$$

Show that L belongs to Σ_2^P.

2 Shallow circuitry [2 points]

Let L be the following language:

$$L = \{\ w \mid\ \text{the string 101 does not appear in } w\ \}.$$

Show that $L \in NC^1$. Also show that L is in AC^0. If you do not want to draw the circuit this is OK, or if you want to draw it by hand and insert this into the homework that is OK as well.

3 The majority gate [1 point]

Here we define a new type of a gate to be used in circuits called a *majority gate*. A majority gate takes n inputs $x_1, \ldots, x_n$(that is, the fan-in is unbounded), and returns the value $(\sum_{i=1}^{n} x_i) \geq \frac{n}{2}$. That is, it returns 1 if and only if at least half of the inputs have the value 1.

In this task you are asked to show that the language L above can be computed by circuits that use only the majority gate and the negation gate. You do not need to draw any circuits if you do not feel inclined to do so. A simple description will do.