PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory, Semester I 2018 - IIC3242
Homework 4 (the easiest one :)
Deadline: Sunday, May 13th, 2018

1 A travelling warmup [2 points]

The travelling salesman problem, denoted TSP, is one of classical NP-complete problems. Recall that TSP is the following language:

$$\text{TSP} = \{\langle G, cost, k\rangle \mid \text{where } G \text{ has a tour whose cost is less than or equal } k\}.$$

Here G is a directed graph, *cost* is a function assigning a non-negative integer (cost) to each edge in G, and k is an integer. A tour in G is a sequence of nodes $\pi = a_1, \ldots, a_n$ such that the list includes all the nodes in G without repetitions, and (a_i, a_{i+1}) is an edge in G, for $i = 1 \ldots n-1$, plus (a_n, a_1) is also an edge in G. The cost of π is calculated as $cost(\pi) = \Sigma_{i=1\ldots n} cost(a_i, a_{i+1}) + cost(a_n, a_1)$.

Show that in the case that TSP can be solved in polynomial time, then, given a graph G and a cost function *cost* as input, we can find one optimal tour in polynomial time.

2 Inefficient problems [1 point]

Define the language U as follows:

$$U = \{\ \langle M, w, \#^t\rangle \mid M \text{ is a non-deterministic TM which accepts } w \text{ within } 2^t \text{ steps, on some branch of its computation }\}.$$

Show that U can not be decided in polynomial time.

3 Impact of inefficient problems on PTIME and NP [3 points]

Show that 2EXPTIME $\neq$ 2NEXPTIME implies that PTIME $\neq$ NP. Recall that 2EXPTIME denotes the class of all languages solvable by a deterministic Turing machine running in time $O(2^{2^{n^c}})$, and similarly for 2NEXPTIME.

Hint: Use the idea from problem 2 of padding the input to a Turing machine with a sufficiently long number represented in unary. try to guess how long this should be for double exponential times.