PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
ESCUELA DE INGENIERIA
DEPARTAMENTO DE CIENCIA DE LA COMPUTACION

Complexity Theory, Semester I 2017 - IIC3242
Homework 4 (a freebie)
Deadline: Tuesday, May 2nd, 2017

1 Completeness in NLOGSPACE [3 points]

If (G, E) is a directed graph, a path $v_1, v_2 \ldots, v_k$ in G is called a *cycle* if $v_1 = v_k$. Let B be the following language:

$$B = \{ \langle G, s \rangle \mid G \text{ is a directed graph, } s \text{ is a node in } G, \text{ and } G \text{ has a cycle that contains } s \}.$$

Show that B is NLOGSPACE-complete. Note that here you need both the upper and the lower bound.

2 The padding technique [4 points]

[a] (1 point) Define the language U as follows:

$$U = \{ \langle M, w, \#^t \rangle \mid M \text{ is a non-deterministic TM which accepts } w \text{ within } t \text{ steps, on some branch of its computation } \}.$$

Show that U is an NP-complete problem. You can assume that # does not appear in the language of M, but this is not important for the problem. The idea of adding a representation of a number in unary at the end of the input is commonly used in complexity theory and is often referred to as *padding*.

[b] (3 points) Using the idea of padding the input to a machine with a representation of a number **in unary**, prove that EXPTIME $\neq$ NEXPTIME implies that PTIME $\neq$ NP.